PlateMesh© Validation Cases

Document Version 1.1 (4/7/2020)

OVERVIEW

This document compares classical solutions to the results from the FEA plate buckling solution. The objective is to demonstrate that the FEA solution is accurate for a variety of load cases, boundary conditions, and material options.

All FEA solutions use 16 elements on the short edge, QUAD8, Reduced Integration, and Allow Stretch set to "N". This is consistent with the "High" option in the FEA program. Based on the convergence study document, these settings demonstrated a converged result for all considered cases. While these settings are associated with a relatively slow solution (and potentially excessive), the primary objective is to best compare the classical solutions to the most accurate FEA solution. The convergence study document assists the user in determining the most computationally efficient solution for their goals.

SUMMARY

A total of 27 cases were considered (various loading, boundary conditions, and materials). For the cases with an ***"exact" classical solution, the agreement was within 2% (most cases were within 1%).*** For cases that showed less agreement, this is likely due to ***limitations of the "non-exact" classical solutions*** (discussed in the following paragraphs). Overall, all 27 cases showed agreement to within 6%.

For the isotropic plate where combined loading was not present, the FEA solutions were *usually within 1% of the classical solutions* (See Table 1). However, the combined loading cases did not show the same level of agreement (up to 5% different) – (See Table 1). This may be because the *classical solutions use interaction equations that may be approximate*. This seems to be a reasonable conclusion because the FEA solutions are very accurate for the "exact" classical solutions (Nx loading, Nxy loading, and Nx,m loading), and this same level of accuracy should carry over to the combined loading cases.

For the single orthotropic load case considered, the agreement was within 1% (See Table 2).

Table 1: Isotropic Plate. Percent difference between the classical solution and the FEA solution.

ISOTROPIC PLATE

Nx Loading

Case	% Diff
1	-0.4
2	-0.2
3	1.4
4	0.9
5	0.3

Nxy Loading

Case	% Diff
1	-0.5
2	1.8
3	0.4
4	1.1

Nx,m Loading

Case	% Diff
1	-0.7
2	0.5

Combined Loading Nxy and Nx,m

Case	% Diff
1	-5.0

Combined Loading Nx and Nx,m

Case	% Diff
1	5.0

Combined Loading Nxy and Nx

Case	% Diff
1	-1.0

Table 2: Orthotropic plate. Percent difference between the classical solution and the FEA solution.

ORTHOTROPIC PLATE

Nx Loading

Case	% Diff
1	-0.4

For the composite laminates, there was good, but less, agreement between the classical solutions and the FEA solution (See Table 3). For Cases 1 through 9, the laminate had non-zero D16 and D26 values. These are not accounted for with the classical solution, but will reduce the Eigenvalue compared to scenario where they are zero. *In other words, the actual Eigenvalue is lower than the predicted by the classical solution*. This is the likely explanation for the lower agreement compared to the isotropic solutions and the lower predictions for the FEA solutions (*the FEA solutions are likely to be more accurate*). For cases 10 through 12, D16=D26=0.0, and the agreement is better for these cases.

Table 3: Composite Laminates. Percent difference between the classical solution and the FEA solution.

COMPOSITE LAMINATES

Nx Loading

Case	% Diff
1	-2.9
2	-4.4
3	-1.1
4	-1.1
5	-3.3
6	-0.5
7	-2.5
8	-5.6
9	-1.0
10	-1.4
11	-1.1
12	-1.9

SOURCES FOR CLASSICAL SOLUTIONS

Source 1: *Practical Stress Analysis for Design Engineers*, Flabel, 2005.

Source 2: *Practical Analysis of Aircraft Composites*, Esp, 2017.

ISOTROPIC MATERIAL

ISOTROPIC MATERIAL – NX LOADING

For all cases, E=10.0E+06, v=0.3, t=0.05

$$N_{x,cr} = K_c \cdot t \cdot \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left[\frac{t}{H}\right]^2$$

Case 1:

All edges SS, W=10.0, H=10.0, W/H=1.0, Kc = 4.0

Classical Soln (Source 1) = -45.2

FEA Soln = -45.0

Difference = -0.4%

Case 2:

All edges SS, W=50.0, H=10.0, W/H=5.0, Kc = 4.0

Classical Soln (Source 1) = -45.2

FEA Soln = -45.1

Difference = -0.2%

Case 3:

T/B=Clamped, L/R = Simple, W=50.0, H=10.0, W/H=5.0, Kc = 6.98

Classical Soln (Source 1) = -78.9

FEA Soln = -80.0

Difference = +1.5%

Case 4:

T =Clamped, L/R/B = Simple, W=50.0, H=10.0, W/H=5.0, Kc = 5.41

Classical Soln (Source 1) = -61.1

FEA Soln = -61.7

Difference = +0.9%

Case 5:

T =Clamped, L/R = Simple, B=Free, W=50.0, H=10.0, W/H=5.0, Kc = 1.28

Classical Soln (Source 1) = -14.46

FEA Soln = -14.5

Difference = +0.3%

ISOTROPIC MATERIAL – NXY LOADING

For all cases, E=10.0E+06, v=0.3, t=0.05

$$N_{xy,cr} = K_s \cdot t \cdot \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left[\frac{t}{H}\right]^2$$

Case 1:

All edges SS, W=10.0, H=10.0, W/H=1.0, Ks = 9.34

Classical Soln (Source 1) = 105.5

FEA Soln = 105.0

Difference = -0.5%

Case 2:

All edges Clamped, W=10.0, H=10.0, W/H=1.0, Ks = 14.58

Classical Soln (Source 1) = 164.7

FEA Soln = 167.7

Difference = +1.8%

Case 3:

All edges SS, W=50.0, H=10.0, W/H=5.0, Ks = 5.50

Classical Soln (Source 1) = 62.1

FEA Soln = 62.4

Difference = +0.4%

Case 4:

All edges Clamped, W=50.0, H=10.0, W/H=5.0, Ks = 9.20

Classical Soln (Source 1) = 103.9

FEA Soln = 105.1

Difference = +1.1%

ISOTROPIC MATERIAL – MOMENT LOADING (NX,M)

For all cases, E=10.0E+06, v=0.3, t=0.05

$$N_{x,m,cr} = K_b \cdot t \cdot \frac{\pi^2 E}{12\left(1 - \nu^2\right)}\left[\frac{t}{H}\right]^2$$

Case 1:

All edges SS, W=10.0, H=10.0, W/H=1.0, Kb = 25.6

Classical Soln (Source 1) = 289.2

FEA Soln = 287.1

Difference = -0.7%

Case 2:

All edges SS, W=10.0, H=50.0, W/H=5.0, Kb = 23.9

Classical Soln (Source 1) = 270.0

FEA Soln = 271.3

Difference = +0.5%

ISOTROPIC MATERIAL – COMBINED SHEAR AND MOMENT

E=10.0E+06, v=0.3, t=0.05

Case 1:

All edges SS, W=10.0, H=10.0, W/H=1.0

Classical Soln (Source 1):

Nxy = 75, Nxy,cr = 105.5, Rs = 0.711

To satisfy $Rs^2+Rb^2 = 1.0$

Rb = 0.703 and Nxm = 203.3 = 0.703*289.2

For the FEA solution, using Nxy = 75 and Nx,m = 203.3:

Eigenvalue = 0.95

Difference = -5.0%

ISOTROPIC MATERIAL – COMBINED COMPRESSION AND MOMENT

E=10.0E+06, v=0.3, t=0.05

Case 1:

All edges SS, W=10.0, H=10.0, W/H=1.0

Classical Soln (Source 1):

Nx = -20, Nx,cr = -45.2, Rc = 0.442

To satisfy $Rb^{1.75}+Rc = 1.0$

Rb = 0.717 and Nxm = 207.4 = 0.717*289.2

For the FEA solution, using Nx = -20 and Nx,m = 207.4:

Eigenvalue = 1.05

Difference = +5.0%

ISOTROPIC MATERIAL – COMBINED COMPRESSION AND SHEAR

E=10.0E+06, v=0.3, t=0.05

Case 1:

All edges SS, W=10.0, H=10.0, W/H=1.0

Classical Soln (Source 1):

Nx = -30, Nx,cr = -45.2, Rc = 0.664

To satisfy Rc +Rs^2 = 1.0

Rs = 0.580 and Nxy = 61.2 = 0.580*105.5

For the FEA solution, using Nx = -30 and Nxy = 61.2:

Eigenvalue = 0.99

Difference = -1.0%

ORTHOTROPIC MATERIAL

ORTHOTROPIC MATERIAL – NX LOADING

Case1:
All edges SS, W=10.0, H=10.0, W/H=1.0
Nx Loading

Ex = 10.0E+06
Ey = 4.0E+06
vxy = 0.35
Gxy = 2.0E+06
Gxz = 2.0E+06
Gyz = 2.0E+06
t = 0.05

From the above values, the [D] matrix properties from the plate can be determined with the eLaminate© program at www.espcompostes.com, as follows:

D11 = 109.5
D22 = 43.9
D12 = 15.3
D66 = 20.8

The classical solution is determined via *Practical Analysis of Aircraft Composites* by Esp, Table 17.4:

Classical Soln: -26.4
FEA Soln: -26.3
Difference = -0.4%

COMPOSITE

COMPOSITE LAMINATE – NX LOADING

For all cases: All edges SS, Nx Loading

Cases 1 to 9 use Material 1 (representative of a unidirectional ply):
Material 1:
E1 = 27.0E+06
E2 = 1.50E+06
v12 = 0.35
G12 = 1.1E+06
G13 = 1.1E+06
G23 = 1.1E+06
t = 0.007

Cases 10 to 12 use Material 2 (representative of a fabric ply):
Material 2:
E1 = 9.0E+06
E2 = 9.0E+06
v12 = 0.05
G12 = 1.0E+06
G13 = 1.0E+06
G23 = 1.0E+06
t = 0.01

Case 1:
Basic Symmetric Layup (Material 1), W=10.0, H=10.0, W/H=1.0
Layup = [0/45/-45/90/90/-45/45/0]

From the layup, the [D] matrix properties from the laminate can be determined with the eLaminate© program at www.espcompostes.com, as follows (D16 is provided for reference):

D_{11} = 281.1
D_{22} = 69.8
D_{12} = 42.3
D_{16} = D_{26} = 17.6
D_{66} = 50.6

The classical solution is determined via *Practical Analysis of Aircraft Composites* by Esp, Table 17.4:

Classical Soln: -62.9
FEA Soln: -61.1
Difference = -2.9%

Case 2:
Same layup as Case 1, W=50.0, H=10.0, W/H=5.0
Same D values and classical solution as Case 1
Classical Soln: -56.8
FEA Soln: -54.3
Difference = -4.4%

Case 3:
Same layup as Case 1, W=4.0, H=10.0, W/H=0.4
Same D values and classical solution as Case 1
Classical Soln: -202.7
FEA Soln: -200.5
Difference = -1.1%

Case 4:

Bias Symmetric Layup (different from Case 1 – Material 1), W=10.0, H=10.0, W/H=1.0
Layup = [0/0/45/-45/-45/45/0/0]
Same classical solution as Case 1

D11 = 363.7
D22 = 35.0
D12 = 18.4
D16 = D26 = 8.8
D66 = 26.7

Classical Soln: -53.5
FEA Soln: -52.9
Difference = -1.1%

Case 5:

Same layup as Case 4, W=50.0, H=10.0, W/H=5.0
Same D values and classical solution as Case 4

Classical Soln: -36.7
FEA Soln: -35.5
Difference = -3.3%

Case 6:

Same layup as Case 4, W=50.0, H=4.0, W/H=0.4
Same D values and classical solution as Case 4

Classical Soln: -239.0
FEA Soln: -237.7
Difference = -0.5%

Case 7:
Unsymmetric Layup (Material 1), W=10.0, H=10.0, W/H=1.0
Layup = [0/45/45/0/0/90/45/-45]
Same classical solution as Case 1

D11 = 202.4
D22 = 108.5
D12 = 62.2
D16 = D26 = 5.9
D66 = 70.6

As discussed in *Practical Analysis of Aircraft Composites* by Esp, Chapter 17, the [D]* matrix properties should be used if the laminate is unsymmetric.

From the eLaminate© program at www.espcompostes.com, the [D]* matrix properties are:

D11* = 171.0
D22* = 73.3
D12* = 42.2
D16* = 9.4
D26* = 23.5
D66* = 59.7

The classical solution is determined via *Practical Analysis of Aircraft Composites* by Esp, Table 17.4 (using D* instead of D):

Classical Soln: -56.0
FEA Soln: -54.6
Difference = -2.5%

Case 8:
Same layup as Case 7, W=50.0, H=10.0, W/H=5.0
Same D* values and classical solution as Case 7

Classical Soln: -54.0
FEA Soln: -51.0
Difference = -5.6%

Case 9:
Same layup as Case 7, W=4.0, H=10.0, W/H=5.0
Same D* values and classical solution as Case 7

Classical Soln: -138.5
FEA Soln: -137.1
Difference = -1.0%

Case 10:
Fabric Symmetric Layup (Material 2), W=10.0, H=10.0, W/H=1.0
Layup = [0/45/0/-45/-45/0/45/0]
Same classical solution as Case 1

D11 = 341.1
D22 = 341.1
D12 = 63.1
D16 = D26 = 0.0
D66 = 86.5

Classical Soln: -113.9
FEA Soln: -112.3
Difference = -1.4%

Case 11:

Same layup as Case 10, W=50.0, H=10.0, W/H=5.0

Same D values and classical solution as Case 10

Classical Soln: -113.9

FEA Soln: -112.7

Difference = -1.1%

Case 12:

Same layup as Case 10, W=50.0, H=10.0, W/H=5.0

Same D values and classical solution as Case 10

Classical Soln: -263.3

FEA Soln: -257.4

Difference = -1.9%